MEMORANDUM

4/27/2026

TO: Eric Ocegueda, Assistant Professor, Mechanical Engineering
FROM: Apolonio Ordaz, ME 328 – 02
SUBJECT: Lab 4: Buckling

Overview

The objective of this lab was to analyze the buckling behavior of a beam under an axial load and how the end conditions have an effect on the critical load the beam can withstand. This was done by designing a solid square cross section of the beam for a 6.5 ft Balsa wood beam. The factor of safety was to remain between 2.5-2.6 while under the different loading conditions and boundary conditions.

Background

Buckling is a failure mode that occurs when a structural member becomes unstable under compressive loading, often at loads lower than the materials yield strength. Eulers buckling theory provides a formula in which the critical load a beam will buckle can be estimated. The equation is as follows:

$$P_{cr} = \frac{\pi^2 CEI}{L^2}$$

Where:

- P_{cr}= Critical Load
- C = End condition constant based
- E= modulus of Elasticity
- I= moment of Inertia
- L= Length of Beam

For a solid square cross section $I = a^4/12$. Substituting this inertia equation into Eulers equation allows to solve for the required side length, a, for the solid square cross section. The constant C can be obtained from Figure 1, which showcases the general deformation profile and constant values for the different boundary conditions.

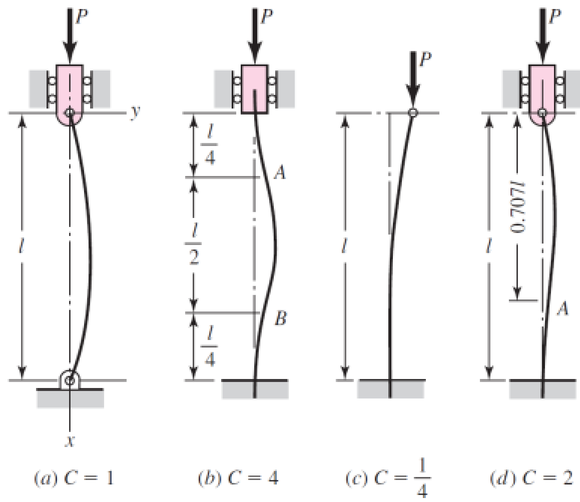


Figure 1. *C* Values and Deformation Profile for (a) pinned-pinned, (b) fixed-fixed, (c) fixed-free, and (d) fixed-pinned

Methods

Hand calculations were performed for each of the four cases using Eulers Buckling equation. The applied load was multiplied by the required factor of safety to determine the necessary critical load. The cross-sectional dimension, *a,* was then solved by using the moment of inertia for a square cross section.

The 4 loading conditions are listed in Table 1 along with the expected hand calculation results at a Factor of Safety of 2.5.

Table 1. Estimated Cross-Section Dimensions from Euler Buckling Theory (Safety Factor = 2.5)

Case	Boundary Condition	Load (lbf)	Estimated Side Length *a* (in.)
1	Pinned-Pinned	22500	5.53
2	Fixed-Pinned	22500	4.65
3	Pinned-Pinned	45000	6.61
4	Fixed-Pinned	45000	5.55

FEA Modeling

A buckling study was conducted in SolidWorks Simulation for each case. Beam elements were used with a mesh size of 0.5 inches were used. The top end of the Balsa Beam remained pinned in all 4 configurations. This was done by restricting the beam in all but the axial direction, while still allowing free rotation. The bottom of the Balsa beam was fixed in all three directions while allowing rotation for the pinned-pinned configurations, while for the fixed-pinned configurations it was restricted in all three directions as well as rotation in all three directions. The loads for each configuration were applied at the top of the Balsa Beam.

The initial cross-sectional dimensions obtained from hand calculations were used as starting values. These were then iteratively adjusted until the resulting load factor from the simulation achieved our desired factor of safety range between 2.5 and 2.6. The final dimensions and achieved factor of safety can be seen in Table 2.

Table 2: Final Cross-Section Dimensions and Safety Factors from FEA Buckling Analysis

Case	Boundary Condition	Load (lbf)	Actual Side Length (in)	Factor of Safety
1	Pinned-Pinned	22500	5.67	2.50
2	Fixed-Pinned	22500	4.75	2.51
3	Pinned-Pinned	45000	6.75	2.52
4	Fixed-Pinned	45000	5.70	2.52

The buckling mode shapes for the pinned–pinned configurations are shown in Figures 2 and 3. In both cases, the column deforms symmetrically about the midpoint, which is the expected behavior for these boundary conditions where both ends are free to rotate. The maximum lateral displacement occurs at the midpoint, indicating that this is the most critical location for instability. As the applied load increases from 22,500 lbf to 45,000 lbf, the deformation profile remains unchanged. However, the cross-sectional dimensions must be increased to maintain the required factor of safety.

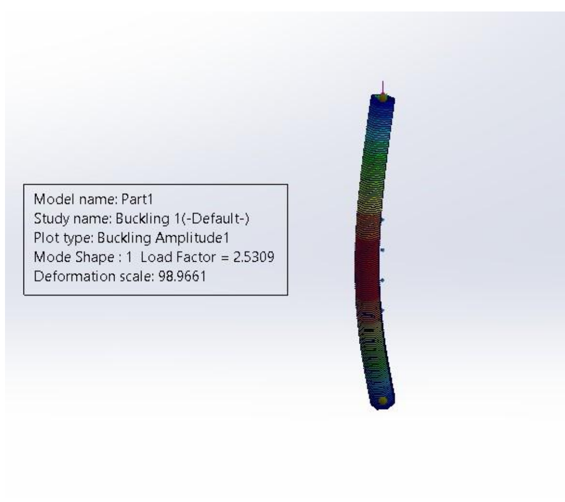


Figure 2. Buckling Mode Shape – Pinned–Pinned, 22,500 lbf (Case 1)

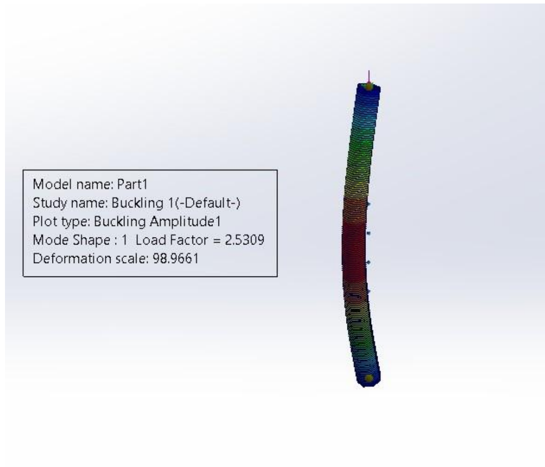


Figure 3. Buckling Mode Shape – Pinned–Pinned, 45,000 lbf (Case 3)

The buckling mode shapes for the fixed–pinned configurations are shown in Figures 4 and 5. Unlike the pinned–pinned cases, the deformation profile is asymmetric due to the fixed support, which restricts both translation and rotation at that end. This results in reduced displacement near the fixed end and larger lateral displacement toward the pinned end. The additional constraint increases the overall stiffness of the column, allowing for a smaller cross section compared to the pinned–pinned configurations. Similar to the previous cases, increasing the applied load does not significantly alter the deformation shape, but it does require a larger cross-sectional area to maintain the desired factor of safety

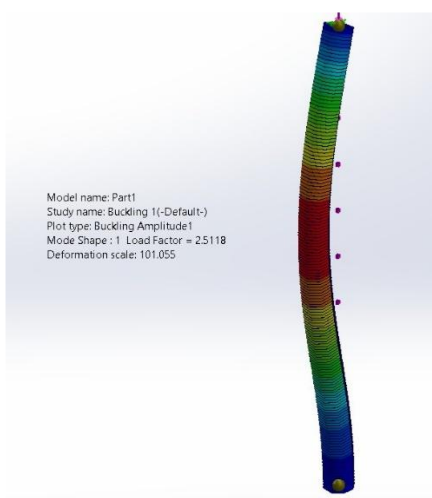


Figure 4. Buckling Mode Shape – Fixed–Pinned, 22,500 lbf (Case 2)

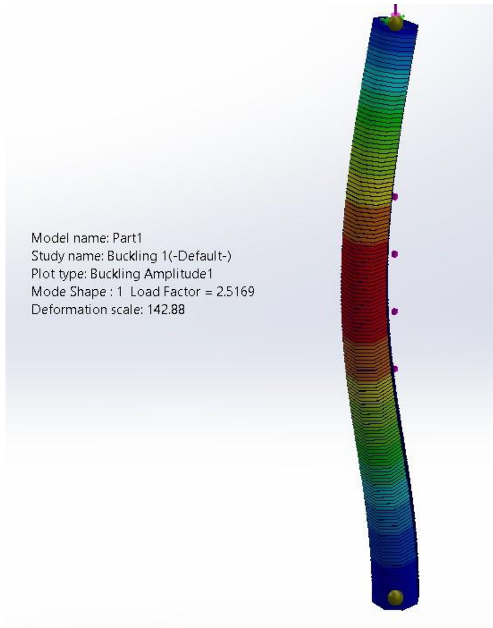


Figure 5. Buckling Mode Shape – Fixed–Pinned, 45,000 lbf (Case 4)

Discussion

Comparison of Hand Calculations and FEA

The hand calculations provided reasonable initial estimates for the required cross-sectional dimensions. However, the calculated values were giving a slightly smaller factor of safety than expected. Thus, the dimension found to reach the required range was slightly larger than calculated, for all cases. These differences can be boiled down to ideal assumptions when utilizing Eulers Theory equations, and the numerical calculations that FEA runs. Improvements could be made by utilizing a smaller element size at the expense of time to run the FEA simulation.

Effect on Boundary Conditions

Boundary conditions had a significant effect on the buckling behavior experienced by the beam. Columns with the pinned-pinned configuration required larger cross sections compared to the fixed-pinned configurations. This can be seen in Eulers equations where the Constant C is larger in the fixed-pined configuration, meaning that the critical load is larger. Intuitively this also makes sense as the pinned-pinned support, both ends of the beam are free to rotate, which lowers the overall stiffness and resistance to movement under a load. As a result, the larger cross section is needed in order to achieve the same safety factor. On the other hand, the fixed-pinned columns have one end that is fully restrained and cannot rotate in a direction. This increases the stiffness of the beam and its resistance to movement under a load also increased.

Effect of Load Magnitude

Higher applied loads required larger cross-sectional dimensions in order to maintain the same fact of safety. However, the increase in size is not proportional because the stiffness of the column depends on the moment of inertia. For a square cross section this increases with the 4^{th} power of the side length. As a result, even a small increase in the cross-sectional dimensions significantly increases the columns resistance to buckling. This effect can be seen in our simulation data as doubling the load did not equate to doubling the cross-sectional dimensions, but rather only an increase of around one inch.

Safety Factor

The safety factor represents the ration of the critical buckling load to the applied load and serves as a measure of how much additional load the beam can withstand before failure. Designing within a narrow range 2.5-2.6 ensured that the structure was both safe and efficient for the conditions of this lab. Having a lower factor of safety risks is instability and potential failure. Conversely having a large factor of safety uses much more resources which drives prices up. It is important to note that the correct safety factor can be subjective but depends on the specific application. Things to consider may include the consequences of failure, variability in materials, loading uncertainty, environmental conditions, and public safety.

Conclusion

This lab demonstrates the application of Euler Buckling theory and finite element analysis in designing stable columns under compressive loading. Hand calculations allowed for an accurate starting point, while FEA allowed the ability to refine and meet the safety requirements. These results emphasize the importance of designing with boundary conditions and loading conditions in mind. Having a good understanding of these design considerations as well as factors of safety allows engineers to design columns as efficiently as possible, while ensuring safety and redundancy in design.

References

Ocegueda, E., *Lab 4: Buckling*, California Polytechnic State University

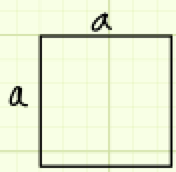
CAL POLY

Appendix A

Pinned-Pinned 22,500 lbf

Hand Calcs

Pinned Pinned $P = 22{,}000 \text{ lbf}$



Need S.F of 2.5

$S.F = \frac{P_{cr}}{P} \rightarrow P_{cr} = 2.5(22{,}000 \text{ lbf})$

$E_{Balsa} = 435113 \frac{\text{lbf}}{\text{in}^2}$

$P_{cr} = 55{,}000 \text{ lbf}$

$L = 6.5 \text{ ft}$

$I = 12 a^4$

$C = 1$

Sub

$$P_{cr} = \frac{C\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 E C a^4}{12 L^2}$$

$$a = \left(\frac{12 P_{cr} L^2}{\pi^2 E C}\right)^{\frac{1}{4}}$$

$$= \left[\frac{(55{,}000 \text{ lbf})(6.5 * 12 \text{ in})^2}{\pi^2 (435113 \frac{\text{lbf}}{\text{in}^2})(12)}\right]^{\frac{1}{4}}$$

$$\boxed{a = 5.529 \text{ inches}}$$

Actual $a = 5.67$

$F.S = 2.5$

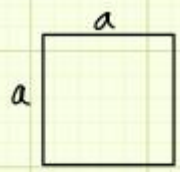
Appendix B

Pinned-Pinned 45,000 lbf

Hand Calcs

Pinned Pinned $P = 45,000\ \text{lbf}$

Need S.F of 2.5



$$S.F = \frac{P_{cr}}{P} \rightarrow P_{cr} = 2.5(45,000\ \text{lbf})$$

$E_{6061} = 435113\ \frac{\text{lbf}}{\text{in}^2}$

$$P_{cr} = 112,500\ \text{lbf}$$

$L = 6.5\ \text{ft}$

$I = 12a^4$

$C = 1$

sub

$$P_{cr} = \frac{C\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 E C a^4}{12\ L^2}$$

$$a = \left(\frac{12 P_{cr} L^2}{\pi^2 E C}\right)^{\frac{1}{4}}$$

$$= \left[\frac{(112,000\ \text{lbf})(6.5 \cdot 12\ \text{in})^2}{\pi^2 (435113\ \frac{\text{lbf}}{\text{in}^2})(12)}\right]^{\frac{1}{4}}$$

$$\boxed{a = 6.605\ \text{inches}}$$

Actual $a = 6.75$

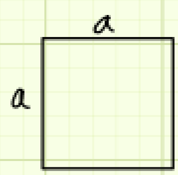
$F.S = 2.5233$

Appendix C

Fixed-Pinned 22,500 lbf

<u>Hand Calcs</u>

Fixed - Pinned $P = 22{,}000\ lbf$



Need S.F of <u>2.5</u>

$S.F = \frac{P_{cr}}{P} \rightarrow P_{cr} = 2.5\,(22{,}000\ lbf)$

$P_{cr} = 55{,}000\ lbf$

$E_{BuGa} = 435113\ \frac{lbf}{in^2}$

$L = 6.5\ ft$

$I = 12\,a^4$

$C = 2$

sub

$$P_{cr} = \frac{C\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 E C a^4}{12\, L^2}$$

$$a = \left(\frac{12 P_{cr} L^2}{\pi^2 E C}\right)^{\frac{1}{4}}$$

$$= \left[\frac{(55{,}000\ lbf)(6.5 \cdot 12\ in)^2}{2 \cdot \pi^2 \left(435113\ \frac{lbf}{in^2}\right)(12)}\right]^{\frac{1}{4}}$$

$$\boxed{a = 4.649\ \text{inches}}$$

Actual

$a = 4.75$ inches

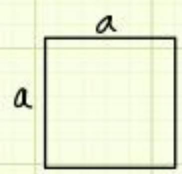
$F.S = 2.5118$

Appendix D

Fixed-Pinned 45,000 lbf

Hand Calcs

Fixed Pinned $P = 45{,}000\ lbf$



Need S.F of 2.5

$$S.F = \frac{P_{cr}}{P} \rightarrow P_{cr} = 2.5\,(45{,}000\ lbf)$$

$$P_{cr} = 112{,}500\ lbf$$

$E_{Birch} = 435113\ \frac{lbf}{in^2}$

$L = 6.5\ ft$

$I = 12\, a^4$

$C = 2$

sub

$$P_{cr} = \frac{C\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 E C a^4}{12\, L^2}$$

$$a = \left(\frac{12 P_{cr} L^2}{\pi^2 E C}\right)^{\frac{1}{4}}$$

$$= \left[\frac{(112{,}000\ lbf)(6.5 \cdot 12\ in)^2}{2 \cdot \pi^2 \left(435113\ \frac{lbf}{in^2}\right)(12)}\right]^{\frac{1}{4}}$$

$$\boxed{a = 5.55\ \text{inches}}$$

Actual

$a = 5.7$

$F.S = 2.5169$